

All of the topics in this booklet are on the Higher GCSE syllabus and are **crucial** to success in A-level Maths. It is therefore essential that you revise, practise and develop your skills in these areas. You can use the links to the website DrFrostMaths.com to help you – you will receive an email inviting you to register and set up a password (if you haven't already done so).



You should complete the booklet assessing your work as you go using the answers at the back of the booklet. We expect to see **full working out for each question** – naturally any questions where the answer has just been copied down will not be seen as completed.

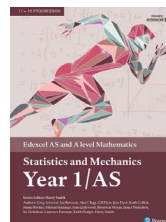
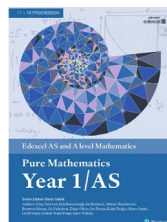
The questions with * will be marked during your first lesson and will indicate to you which areas of Maths you need to ask for help with before the **baseline assessment**, which will take place in early September. Please complete these on a separate piece of paper with your name clearly written so that you can hand this work to your teacher on the first lesson.

Extra help is available at www.pearsonactivelearn.com where you can log on using

Username: Induction@StBenedicts

Password: sbsjAlevel23

This will give you access to the two textbooks that form the basis of the Y12 course and you are encouraged to familiarise yourself with them.



Next to each exercise below (where applicable) there is a link to a video tutorial on DR Frost.



Section 1 – Algebraic Expressions

1. Simplify as far as possible

a. $3x - 2y + 4y$

b. $7 - 3x + 2 + 4x$ *

c. $5x + 2y - 4y - x^2$

2. Factorise as fully as possible

a. $x^2 + 5x$

b. $2y^2 + 6y$

c. $acb - ab^3$ *

d. $ax + ay + bx + by$

3. Multiply out the brackets and simplify

a. $3x + 2(x + 1)$

b. $5x - 2x(x - 1)$

c. $3x(x - 1) - 7x^2$ *

d. $(x - 3)(x - 5)$

e. $(2x + y)(x - 3y)$

Expanding & Removing Brackets

<https://youtu.be/QwAwH5qiA1Y>

4. Simplify the following

a. $\frac{6y}{3y}$

b. $\frac{4ab^2c^3}{3a^3b^2c}$ *

Simplifying Algebraic Fractions

<https://youtu.be/PMXJGwzW1AM>

5. Simplify the following

a. $\frac{3x}{2} \times \frac{2a}{3x}$ *

b. $\frac{5ab^2}{2} \times \frac{3}{2a^2b}$

6. Simplify the following

a. $\frac{2}{a} \div \frac{a}{2}$

b. $\frac{3(x-1)}{x} \div \frac{6}{xy}$ *

7. Write as a single fraction

a. $\frac{x}{2} + \frac{x+1}{3}$ *

b. $\frac{3(x-1)}{x} - \frac{6}{xy}$

Adding and subtracting Algebraic Fractions

<https://youtu.be/PQWQORRpUIQ>



Section 2 – Linear Equations

Solve the following equations, give your answers as fractions in mixed number form where necessary.

1. $2x - 5 = 11$

2. $12 = 2x - 8$

3. $\frac{3x}{2} = 5$

4. $4 = \frac{x}{2} + 5$

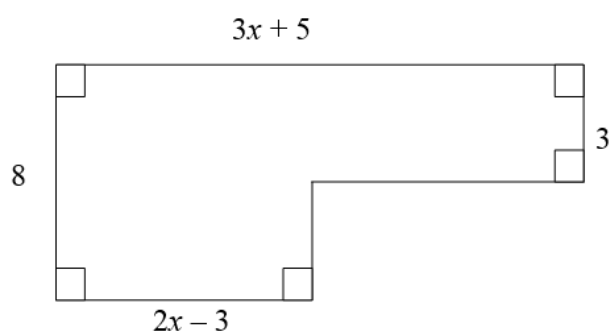
5. $x + 3(x + 1) = 2x$

6. $\frac{1}{3}(x + 2) = \frac{1}{5}(3x + 2)$

7. $\frac{2x-1}{3} = \frac{x}{2} \quad *$

Linear Equations in one variable <https://youtu.be/l-za234E9F4> <https://youtu.be/IHgy9iAk-wU>

9. The area of this shape is 38 cm^2 .
All the measurements are in cm.
Find the length of the smallest side.



Section 3 - Formulae

Make x the subject of each of the following formulae

1. $2x = S$

2. $9x = T + N$

3. $V^2x + b = c$

4. $3(x - 1) = a$

5. $\frac{B-Ax}{D} = E$

6. $\frac{Q}{x-C} = T \quad *$

Rearranging Formulae <https://youtu.be/y-cGpOFCB3Q>

7. $\frac{m^2}{x} - n = -p$

8. $g = \sqrt{c - x}$

9. $t = \sqrt{(x^2 - k)} - b \quad *$

10. $zx = S(x + t)$

11. $x(a - c) = bx + d$

12. $\sqrt{(x^2 + a^2)} = b^2$



Section 4 – Simultaneous Equations (both linear)

Solve the following pairs of simultaneous equations:

1. $2x + 5y = 24$

$$4x + 3y = 20$$

2. $3x - y = 9$
 $4x - y = -14$ *

<https://youtu.be/XWNropJ5pn0>

3. $5x - 7y = 27$
 $3x - 4y = 16$

Section 5 – Factorising Quadratics, x^2 (multiplied by 1)

Factorise each of the following quadratics

1. $x^2 + 7x + 10$

2. $x^2 + 8x + 15$

3. $x^2 - 8x + 16$

4. $x^2 - x - 12$

5. $x^2 - 5x - 24$ *

6. $x^2 - 49$

7. $x^2 + 5x$

<https://youtu.be/5p79yczPWQc>

Section 6 – Factorising Quadratics, x^2 (not multiplied by 1)

Factorise each of the following quadratics

1. $2x^2 + 5x + 3$

2. $3x^2 + 7x + 2$

3. $3x^2 - 11x + 6$

4. $3x^2 - 5x - 2$

5. $2x^2 + x - 21$

6. $12x^2 + 4x - 5$

7. $6x^2 - 27x + 30$ *

8. $4x^2 - 25$

<https://youtu.be/cbbEFYpeWV8>

Section 7 – Completing the Square

Write each of the following in the form $(x + a)^2 + b$ where a and b are constants to be determined:

1. $x^2 + 4x + 7$ *

2. $x^2 - 26x - 1$

3. $x^2 + 5x - 5$

4. $x^2 - x + 1$

<https://youtu.be/sQooemMboCw>

Write each of the following in the form $a(x + b)^2 + c$ where a , b and c are constants to be determined:

5. $2x^2 - 4x + 5$ *

6. $3x^2 - 12x + 10$

7. $5x^2 + 25x - 5$



Section 8 – Solving Quadratic Equations – By Factorising

Solve each of the following

1. $x^2 + 7x + 12 = 0$

2. $x^2 - 8x + 12 = 0$

3. $x^2 + 2x - 15 = 0$ *

4. $6x^2 - 13x + 6 = 0$

Quadratic Equations https://youtu.be/ldk4R_baULA

5. $10x^2 - x - 3 = 0$

6. $6x^2 + 17x - 3 = 0$

7. $9x^2 + 6x = 0$

8. $4x^2 - 1 = 0$

Section 9 – Solving Quadratic Equations - Formula

Solve the following quadratics, giving your solutions correct to 3 significant figures:

1. $x^2 + 7x + 5 = 0$

2. $10x^2 + 3x - 2 = 0$

3. $5x^2 - 9x + 2 = 0$ *

4. $6x^2 - 5x = 8$

Use the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

<https://youtu.be/q0vb8ybymKI>

Section 10 – Equations with Quadratic Solutions

Solve each of the following equations

1. $x + 5 = \frac{14}{x}$

2. $\frac{3}{x-1} + \frac{3}{x+1} = 4$ *

3. $\frac{2}{x-1} - \frac{1}{x+2} = \frac{-4}{x+3}$

Hint: Form and solve a quadratic Equation



Section 11 – Simultaneous Equations (when one is non-linear)

Solve the following pairs of simultaneous equations:
 $y = x^2 - 4x + 6$
 $y = 2x^2 - 3x + 4$

https://youtu.be/2_n1KKPzmm8

1. $y = x - 3$
 $y^2 + xy + 4x = 7$ *

5. $y + 2x = 3$
 $y^2 + xy = 13 - 16x$

2. $x + y = 5$
 $3xy = 18$

6. $3y - 2x = 11$
 $xy = 2$

Example/ Advice.

- If both equations have the same letter as their subject (i.e. Q1) then put them equal to each other, and then solve.
- Otherwise, substitute the linear equation into the non-linear equation, and then solve.

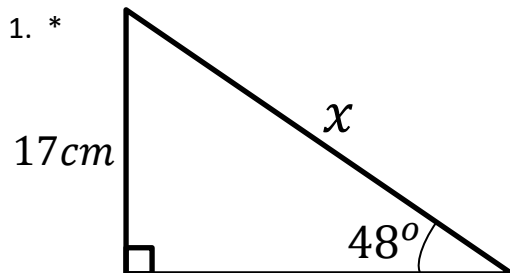
Section 12 – Trigonometry (calculator allowed)

1

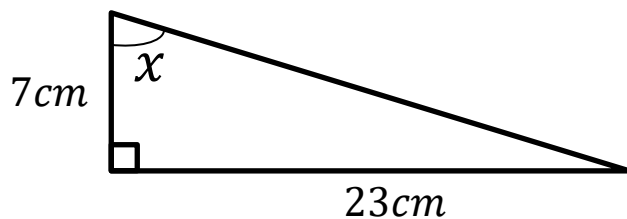
Finding sides <https://youtu.be/r6xuUjwyKek>

Finding angles <https://youtu.be/5brkGmejvqU>

In each diagram, find the value of:



2.



Section 13 - Indices

Laws of indices https://youtu.be/Z_WCKeO-liY

Fractional powers <https://youtu.be/E-gg8eq105I>

Negative powers <https://youtu.be/RBPbRrHSCVs>

1. Simplify:

a) $2p^2 \times 4p^5$ b) $a^2 \times a^3 \times a^5$ c) $12n^7 \div 2n^2$ d) $(y^3)^4$ e) f) $w^4y^3 \div wy^2$

2. Simplify

a) $a^8 \times a^{-6}$ b) $(wk^{-4})^3 *$ c) $2b^{\frac{2}{3}} \times 4b^{\frac{1}{4}}$ d) $\frac{y^2 \times y^{\frac{1}{3}}}{y}$ e) $\frac{4x^{\frac{2}{3}} \times 3x^{-\frac{1}{6}}}{6x^{\frac{3}{4}}} *$

Section 14 - Surds

Simplify https://youtu.be/LDH_VS1fwYE

Add/subtract <https://youtu.be/O0Hm8oAgrPI>

Multiply/divide <https://youtu.be/XiuLvyXyRxY>

Rationalise <https://youtu.be/wiBovf97ihc>

1. Simplify

a) $\sqrt{50}$ b) $\sqrt{98}$ c) $\sqrt{27}$ d) $\sqrt{363}$
e) $\sqrt{18} + \sqrt{50}$ f) $2\sqrt{8} + \sqrt{72} *$ g) $\sqrt{3}(2 + \sqrt{3})$ h) $(3\sqrt{3} + 1)(2 - 5\sqrt{3})$

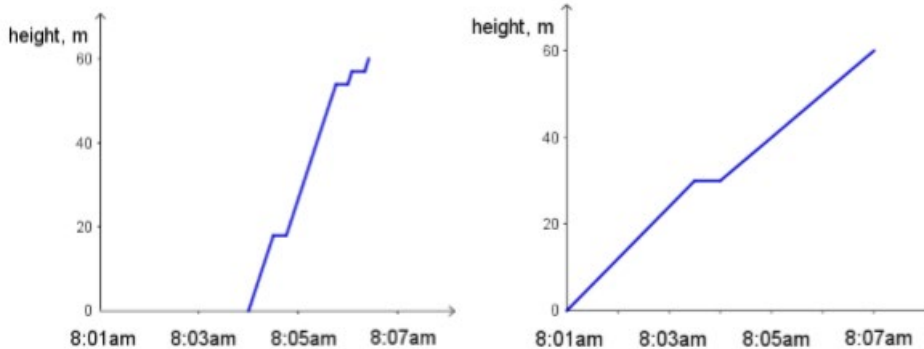
2. Express as simply as possible with a rational denominator

a) $\frac{1}{\sqrt{8}}$ b) $\frac{\sqrt{5}}{\sqrt{15}}$ c) $\frac{4\sqrt{20}}{3\sqrt{18}} *$ d) $\frac{6-\sqrt{8}}{\sqrt{2}}$ e) $\frac{2}{\sqrt{3}} - \frac{\sqrt{6}}{\sqrt{12}} *$



Problem Solving *

1. At a coffee shop Americanos cost £1.90. Cappuccinos cost £2.40. Maddie bought two Americanos and some cappuccinos. She paid with a £20 note and received £9.00 change. How many cappuccinos did she buy? *
2. Jamila has taken five out of six tests. The maximum mark in each test is 20. So far her average (mean) mark is 12. Is it possible for her to increase her average mark to above 15 after she has taken the final test? *
3. Two work colleagues arrive on the ground floor of a building at 8.01 am for a meeting on the twentieth floor. The twentieth floor is 60 m above ground level. One decides to take the stairs, the other the lift.



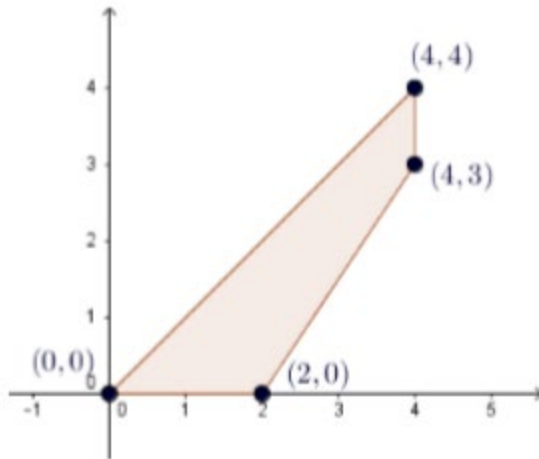
They both arrive on the twentieth floor at 8.07 am. In the meeting room they sketch graphs showing their height above the ground in terms of time between 8.01 am and 8.07 am.

Which graph do you think shows the person who took the lift? Why ?

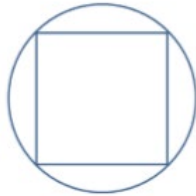
What assumptions might have been made in drawing these graphs? *



4. Calculate the shaded area in the diagram below: *

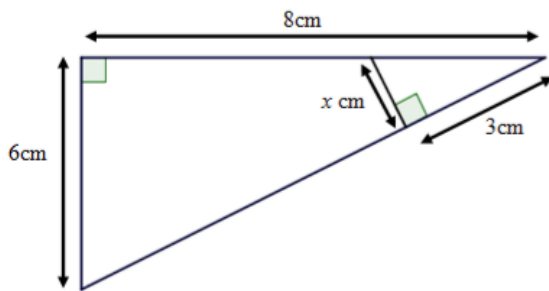


5. The diagram shows a square and a circle. The corners of the square are on the circle. The perimeter of the square is 20 cm.



Find the exact value of the area of the circle. *

6. Find the value of x in the diagram below. *



Further Maths

Any students hoping to study **Further Maths** alongside Maths should also complete the following questions, showing all steps of your method clearly.

1. Simplify these expressions as far as possible.

a. $\frac{x^2 - x - 6}{x^2 + x - 12}$

b. $\frac{x^2 - 49}{x^2 - x - 30} \div \frac{x^2 + 5x - 14}{x^2 - 36}$

2. a. Show that the line $x + y = 6$ is a tangent to the circle $x^2 + y^2 = 18$.

a. Show the line and the circle on a diagram.

b. The tangent meets the x axis at P and the y axis at Q. Find the area of the triangle OPQ.

3. Expand and simplify $(5a - 3\sqrt{c})(5\sqrt{a} - \sqrt{c})$

4. a Write $4x^2 + 24x + 11$ in the form $a(x + b)^2 + c$

b. Hence, or otherwise, write down the coordinates of the turning point of the graph of $y = 4x^2 + 24x + 11$.

5. Prove algebraically that the sum of the squares of three consecutive numbers less 2 is always 3 times a square number.

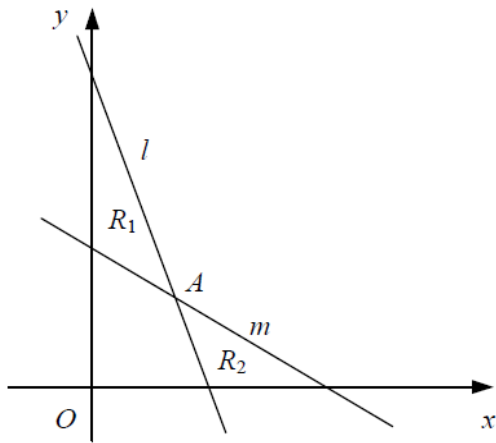
6. The functions f and g are defined as $g(x) = \frac{3x}{2-x}$ and $f(x) = 4x + 1$.

$$2-x$$

Given that $x \neq 2$, find the value(s) of x such that $g(x) = f(x)$, giving your answer(s) to 2 decimal places.



7.



The line l with equation $3x + y - 9 = 0$ intersects the line m with equation $2x + 3y - 12 = 0$ at the point A as shown in the diagram above.

a Find, as exact fractions, the coordinates of the point A .

The region R_1 is bounded by l , m and the y -axis.

The region R_2 is bounded by l , m and the x -axis.

b Show that the ratio of the area of R_1 to the area of R_2 is $25 : 18$

8. A triangle has side lengths $AB = 18$ cm, $BC = 15$ cm and $AC = 8$ cm.

a. Find the size of the largest angle, giving your answer to 2 decimal places.

b. Find the area of the triangle, giving your answer to 2 decimal places.

9. a. Sketch the graph of $y = \sin x$ for $-180^\circ \leq x \leq 360^\circ$, showing the points where the graph cuts the axes.

b. Hence find the exact values of x in the interval $-180^\circ \leq x \leq 360^\circ$ for which

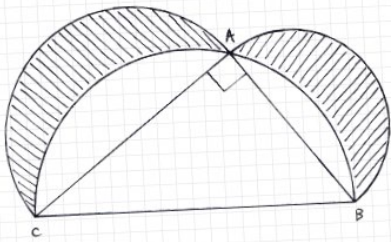
$$\sin x = -\frac{1}{2}$$



10. Problem Solving challenge

NO 1 IN A SERIES OF SIX

My favourite problem is...



This problem looks complicated but requires only a few simple techniques to reveal a remarkable solution and a 2500 year old theorem.

Triangle ABC has a right-angle at A.
Semi-circles are drawn with BA, AC and BC as diameters as shown.
Given that $AC = 8$ and $BC = 10$, write down the value of one third of the total shaded area.

From Senior Team Mathematics Challenge (UKMT/FMSP)

Can you find the solution ?



Answers

Section 1

- 1a) $3x + 2y$ c) $-x^2 + 5x - 2y$
- 2a) $x(x + 5)$ b) $2y(y + 3)$ d) $a(x + y) + b(x + y) = (a + b)(x + y)$
- 3a) $5x + 2$ b) $7x - 2x^2$ d) $x^2 - 8x + 15$ e) $2x^2 + 3y^2 - 5xy$
- 4a) 2
- 5a) b) $\frac{15b}{4a}$
- 6a) $\frac{4}{a^2}$
- 7a) b) $\frac{3xy - 3y - 6}{xy}$ or $\frac{3(xy - y - 2)}{xy}$

Section 2

1. $x = 8$ 2. $x = 10$ 3. $x = \frac{10}{3} = 3\frac{1}{3}$ 4. $x = -2$
5. $x = -\frac{3}{2} = -1\frac{1}{2}$ 6. $x = 4$ 8. $x = 0$

Section 3

1. $x = \frac{S}{2}$ 2. $x = \frac{T + N}{9}$ 3. $x = \frac{c-b}{V^2}$ 4. $x = \frac{a}{3} + 1$ or $\frac{a+3}{3}$
5. $x = \frac{B-ED}{A}$ or $\frac{ED-B}{-A}$ 7. $x = \frac{m^2}{n-p}$ 8. $x = c - g^2$
10. $x = \frac{St}{z-S}$ 11. $\frac{d}{a-c-b}$ 12. $x = \sqrt{b^4 - a^2}$

Section 4

1. $x = 2, y = 4$ 3. $x = 4, y = -1$

Section 5

1. $(x + 2)(x + 5)$ 2. $(x + 3)(x + 8)$ 3. $(x - 4)^2$ 4. $(x - 4)(x + 3)$
6. $(x - 7)(x + 7)$ 7. $x(x + 5)$

Section 6

1. $(2x + 3)(x + 1)$ 2. $(3x + 1)(x + 2)$ 3. $(3x - 2)(x - 3)$ 4. $3(2x - 5)(x - 2)$
5. $(3x + 1)(x - 2)$ 6. $(2x + 7)(x - 3)$ 8. $(2x - 5)(2x + 5)$



Section 7

2. $(x - 13)^2 - 170$ 3. $(x + 2\frac{1}{2})^2 - 11\frac{1}{4}$ 4. $(x - \frac{1}{2})^2 + \frac{3}{4}$
6. $3(x - 2)^2 - 2$ 7. $5(x + 2\frac{1}{2})^2 - 36\frac{1}{4}$

Section 8

1. $x = -4$ or -3 2. $x = 2$ or 6 4. $x = \frac{2}{3}$ or $\frac{3}{2}$
5. $x = \frac{3}{5}$ or $-\frac{1}{2}$ 6. $x = \frac{1}{6}$ or -3 7. $x = 0$ or $-\frac{2}{3}$ 8. $x = \frac{1}{2}$ or $-\frac{1}{2}$

Section 9

1. $x = -6.19$ or -0.807 2. $x = -0.622$ or 0.322
4. $x = -0.811$ or 1.64

Section 10

1. $x = -7$ or 2 3. $x = -\frac{7}{5}$ or -1

Section 11 – (with workings out for the questions 1-3)

1. $x^2 - 4x + 6 = 3x - 4$
 $x^2 - 7x + 10 = 0$
 $(x - 2)(x - 5) = 0$
 $x = 2$ or 5
 $\therefore y = 2$ or 11

 $\rightarrow x = 2, y = 2$ or $x = 5, y = 11$

3. $x = 5 - y \Rightarrow 3(5 - y)y = 18$
 $15y - 3y^2 = 18$
 $3y^2 - 15y + 18 = 0$
 $y^2 - 5y + 6 = 0$
 $(y - 3)(y - 2) = 0$
 $y = 3$ or 2
 $\therefore x = 2$ or 3
 $\rightarrow x = 2, y = 3$ or $x = 3, y = 2$

4. $x = \frac{1}{2}, y = 3$ or $x = 3, y = 13$ 5. $x = \frac{1}{2}, y = 2$ or $x = -4, y = 11$

6. $x = \frac{1}{2}, y = 4$ or $x = -6, y = -\frac{1}{3}$



Section 12

2. $x = 73.07248694 = 73.1^0$ (3 s.f.)

Section 13

1a $8p^7$ 1b) a^{10} 1c) $6n^5$ 1d) y^{12} 1e) w^3y

2a) a^2 2b) w^3k^{-12} 2c) $8b^{\frac{11}{12}}$ 2d) $y^{-\frac{1}{6}}$ 2e) $2x^{-\frac{1}{4}}$

Section 14

1a) $5\sqrt{2}$ 1 b) $7\sqrt{2}$ 1 c) $3\sqrt{3}$ 1 d) $3\sqrt{11}$

1 e) $8\sqrt{2}$ 1 f) $10\sqrt{2}$ 1g) $2\sqrt{3} + 3$ 1 h) $-43+\sqrt{3}$

